Second Year Higher Secondary Model Examination, February 2023
Mathematics (Science)

Answer Key

Answer any 6 questions . Each carries 3 scores.

1. $A=\begin{bmatrix} a_{11} & a_{12} & a_{13} \\ a_{21} & a_{22} & a_{23} \end{bmatrix}$ (3)

$a_{11}=2-2=0$ $a_{21}=4-2=2$

$a_{12}=2-4=-2$ $a_{22}=4-4=0$

$a_{13}=2-6=-4$ $a_{23}=4-6=-2$

$A=\begin{bmatrix} 0 & -2 & -4 \\ 2 & 0 & -2 \end{bmatrix}$

2. $R=\{(1,3),(2,6),(3,9),(4,12)\}$ (3)

R is not reflexive since $(a,a)\notin R$

for every $a\in A$

R is not symmetric since $(1,3)\in R$

but $(3,1)\notin R$.

R is not transitive since $(1,3),(3,9)\in R$

but $(1,9)\notin R$.

3. $2A=2\begin{bmatrix} 1 & 2 \\ 4 & 2 \end{bmatrix}=\begin{bmatrix} 2 & 4 \\ 8 & 4 \end{bmatrix}$ (3)

$|2A|=8-32=-24$

$|A|=2-8=-6$

$4|A|=4(-6)=-24$

$|2A|=4|A|$

4. f(x) is continuous at a, then LHL=RHL=f(a) (3)

f(x) is continuous at x= 2

$LHL=\lim_{x\to 2^-} f(x)=\lim_{x\to 2^-}(5)=5$

$RHL=\lim_{x\to 2^+} f(x)=\lim_{x\to 2^+}(ax+b)=2a+b$

Therefore $2a+b=5$

f(x) is continuous at x= 10

$LHL=\lim_{x\to 10^-} f(x)=\lim_{x\to 10^-}(21)=21$

$RHL=\lim_{x\to 10^+} f(x)=\lim_{x\to 10^+}(ax+b)=10a+b$

Therefore $10a+b=21$

subtracting them , we get $8a=16$, $a=2$

substituting , then $b=5-2a$, $b=1$

5. $f(x)=x^2-4x$ (3)

$f'(x)=2x-4$

$f'(x)=0\Rightarrow 2x=4$ x = 2

So R divides into two intervals (-∞, 2] and [2, ∞)

$f'(-1)=2(-1)=-2<0$

f(x) is decreasing on (-∞, 2]

$f'(3)=2(3)-4=2>0$

f(x) is increasing on [2, ∞)

6. A unit vector in the direction of $\vec{a}$

is given by $\hat{a}=\frac{\vec{a}}{|\vec{a}|}$ (3)

$|\vec{a}|=\sqrt{1^2+1^2+2^2}=\sqrt{1+1+4}=\sqrt{6}$

$\hat{a}=\frac{\hat{i}+\hat{j}+2\hat{k}}{\sqrt{6}}=\frac{1}{\sqrt{6}}\hat{i}+\frac{1}{\sqrt{6}}\hat{j}+\frac{2}{\sqrt{6}}\hat{k}$

7. Angle between two lines is given by,

$\cos\theta=\left|\frac{\vec{b_1}.\vec{b_2}}{|\vec{b_1}||\vec{b_2}|}\right|$ (3)

$\vec{b_1}=\hat{i}+2\hat{j}+2\hat{k}$, $\vec{b_2}=3\hat{i}+2\hat{j}+6\hat{k}$

$|\vec{b_1}|=\sqrt{1^2+2^2+2^2}=\sqrt{9}=3$

$|\vec{b_2}|=\sqrt{3^2+2^2+6^2}=\sqrt{49}=7$

$\vec{b_1}.\vec{b_2}=1.3+2.2+2.6=19$

$\cos\theta=\left|\frac{19}{3.7}\right|=\frac{19}{21}$

$\theta=\cos^{-1}\left(\frac{19}{21}\right)$

8. i) $P(A/B)=\frac{P(A\cap B)}{P(B)}=\frac{P(A).P(B)}{P(B)}=P(A)$

Therefore $P(A/B)=P(A)=0.3$ (1)

ii) $P(A\cap B')=P(A).P(B')$ (2)

since A and B are independent events.

$P(B')=1-P(B)=1-0.6=0.4$

Now , $P(A\cap B')=0.3x\,0.4=0.12$

Answer any 6 questions. Each carries 4 scores.

9. i) y (1)

ii) If $f(x_1)=f(x_2)$ for all $x_1,x_2\in R$ (3)

then $3-4x_1=3-4x_2$

ie, $4x_1 = 4x_2 \quad x_1 = x_2$ for all $x_1, x_2 \in R$

Therefore f(x) is one-one.

Let $y = 3 - 4x$

then $x = \frac{3-y}{4} \in R$ for all $y \in R$

Therefore f(x) is onto.

So f(x) is bijective.

10. i) x (1)

ii) $\frac{\pi}{4}$ (1)

iii) $\sin^{-1}\left(\sin\frac{13\pi}{6}\right) = \sin^{-1}\left(\sin\left(2\pi + \frac{\pi}{6}\right)\right)$ (2)

$= \sin^{-1}\left(\sin\frac{\pi}{6}\right) = \frac{\pi}{6}$

11. $A = \frac{1}{2}(A + A') + \frac{1}{2}(A - A')$ (4)

$$A' = \begin{bmatrix} 3 & -2 & -4 \\ 3 & -2 & -5 \\ -1 & 1 & 2 \end{bmatrix}$$

$$A + A' = \begin{bmatrix} 6 & 1 & -5 \\ 1 & -4 & -4 \\ -5 & -4 & 4 \end{bmatrix}$$

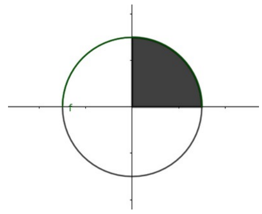
$$\frac{1}{2}(A + A') = \begin{bmatrix} 3 & \frac{1}{2} & \frac{-5}{2} \\ \frac{1}{2} & -2 & -2 \\ \frac{-5}{2} & -2 & 2 \end{bmatrix} \text{ ——— P}$$

$$A - A' = \begin{bmatrix} 0 & 5 & 3 \\ -5 & 0 & 6 \\ -3 & -6 & 0 \end{bmatrix}$$

$$\frac{1}{2}(A - A') = \begin{bmatrix} 0 & \frac{5}{2} & \frac{3}{2} \\ \frac{-5}{2} & 0 & 3 \\ \frac{-3}{2} & -3 & 0 \end{bmatrix} \text{ ——— Q}$$

A = P + Q

12. (4)



Area of the region = $4 \int_0^a \sqrt{a - x^2}\, dx$

$= 4 \left[\frac{x}{2}\sqrt{a^2 - x^2} + \frac{a^2}{2}\sin^{-1}\frac{x}{a}\right]_0^a$

$= 4 \left[\frac{a}{2}\sqrt{a^2 - a^2} + \frac{a^2}{2}\sin^{-1}\frac{a}{a} - 0\right]$

$= 4 \left[\frac{a^2}{2}\sin^{-1} 1\right]$

$= 4 . \frac{a^2}{2} . \frac{\pi}{2}$

$= \pi a^2$ sq. units

13. $\frac{dy}{dx} + \frac{y}{x} = x^2 \quad , \quad P = \frac{1}{x}, Q = x^2$ (4)

$I.F = e^{\int P\,dx} = e^{\int \frac{1}{x}dx} = e^{\log x} = x$

General equation is given by

$y . IF = \int (Q . IF)\, dx + C$

$y . x = \int (x^2 . x)\, dx + C$

$y . x = \int x^3\, dx + C$

$yx = \frac{x^4}{4} + C$

14. Area of the parallelogram = $|\vec{a} \times \vec{b}|$ (4)

$$\vec{a} \times \vec{b} = \begin{vmatrix} \hat{i} & \hat{j} & \hat{k} \\ 3 & 1 & 4 \\ 1 & -1 & 1 \end{vmatrix}$$

$= \hat{i}(1 - -4) - \hat{j}(3 - 4) + \hat{k}(-3 - 1)$

$= 5\hat{i} + \hat{j} - 4\hat{k}$

$|\vec{a} \times \vec{b}| = \sqrt{5^2 + 1^2 + 4^2} = \sqrt{25 + 1 + 16} = \sqrt{42}$ sq. Units

15. Distance between two lines

$$d = \left|\frac{(\vec{b}_1 \times \vec{b}_2).(\vec{a}_2 - \vec{a}_1)}{|\vec{b}_1 \times \vec{b}_2|}\right| \quad (4)$$

$\vec{a}_1 = \hat{i} + 2\hat{j} + \hat{k}$

$\vec{a}_2 = 2\hat{i} + \hat{j} - \hat{k}$

$\vec{b}_1 = \hat{i} - \hat{j} + \hat{k}$

$\vec{b}_2 = 2\hat{i} + \hat{j} + 2\hat{k}$

$\vec{a}_2 - \vec{a}_1 = \hat{i} - \hat{j} - 2\hat{k}$

$$\vec{b}_1 \times \vec{b}_2 = \begin{vmatrix} \hat{i} & \hat{j} & \hat{k} \\ 1 & -1 & 1 \\ 2 & 1 & 2 \end{vmatrix}$$

$= \hat{i}(-2-1) - \hat{j}(2-2) + \hat{k}(1--2)$

$= -3\hat{i} + 3\hat{k}$

$|\vec{b}_1 \times \vec{b}_2| = \sqrt{3^2+3^2} = \sqrt{9+9} = \sqrt{18}$

$$d = \left| \frac{(-3\hat{i}+3\hat{k}).(\hat{i}-\hat{j}-2\hat{k})}{\sqrt{18}} \right|$$

$$= \left| \frac{-3-6}{\sqrt{18}} \right|$$

$$= \frac{9}{\sqrt{18}} = \frac{3}{\sqrt{2}}$$

16. E_1 = Bag I (4)

E_2 = Bag II

A = Red ball, B = Black ball

$$P(E_2/B) = \frac{P(E_2)P(B/E_2)}{P(E_1)P(B/E_1) + P(E_2)P(B/E_2)}$$

$$= \frac{\frac{1}{2}.\frac{6}{8}}{\frac{1}{2}.\frac{4}{8} + \frac{1}{2}.\frac{6}{8}} = \frac{\frac{3}{8}}{\frac{2}{8}+\frac{3}{8}} = \frac{3}{5}$$

Answer any 3 questions. Each carries 6 scores.

17. AX = B (6)

Then $X = A^{-1}B$

And $A^{-1} = \frac{1}{|A|} adj\, A$, $|A| \neq 0$

$$\begin{bmatrix} 1 & -1 & 1 \\ 2 & 1 & -3 \\ 1 & 1 & 1 \end{bmatrix} \begin{bmatrix} x \\ y \\ z \end{bmatrix} = \begin{bmatrix} 4 \\ 0 \\ 2 \end{bmatrix}$$

$$|A| = 1\begin{vmatrix} 1 & -3 \\ 1 & 1 \end{vmatrix} - -1\begin{vmatrix} 2 & -3 \\ 1 & 1 \end{vmatrix} + 1\begin{vmatrix} 2 & 1 \\ 1 & 1 \end{vmatrix}$$

$= 1(1--3) + 1(2--3) + 1(2-1)$

$= 1(4) + 1(5) + 1(1) = 10$

1	-1	1	1	-1	1
2	1	-3	2	1	-3
1	1	1	1	1	1
1	-1	1	1	-1	1
2	1	-3	2	1	-3
1	1	1	1	1	1

$$\text{Co-factor matrix} = \begin{bmatrix} (1--3) & (-3-2) & (2-1) \\ (1--1) & (1-1) & (-1-1) \\ (3-1) & (2--3) & (1--2) \end{bmatrix}$$

$$= \begin{bmatrix} 4 & -5 & 1 \\ 2 & 0 & -2 \\ 2 & 5 & 3 \end{bmatrix}$$

Adj (A) =Transpose of co-factor matrix

$$= \begin{bmatrix} 4 & 2 & 2 \\ -5 & 0 & 5 \\ 1 & -2 & 3 \end{bmatrix}$$

$$A^{-1} = \frac{1}{|A|} adj\, A$$

$$= \frac{1}{10}\begin{bmatrix} 4 & 2 & 2 \\ -5 & 0 & 5 \\ 1 & -2 & 3 \end{bmatrix}$$

$X = A^{-1}B$

$$= \frac{1}{10}\begin{bmatrix} 4 & 2 & 2 \\ -5 & 0 & 5 \\ 1 & -2 & 3 \end{bmatrix}\begin{bmatrix} 4 \\ 0 \\ 2 \end{bmatrix}$$

$$= \frac{1}{10}\begin{bmatrix} 16+0+4 \\ -20+0+10 \\ 4+0+6 \end{bmatrix}$$

$$= \frac{1}{10}\begin{bmatrix} 20 \\ -10 \\ 10 \end{bmatrix} = \begin{bmatrix} 2 \\ -1 \\ 1 \end{bmatrix}$$

$x = 2, y = -1, z = 1$

18. i) $\frac{d}{dx}(4x - 5y) = \frac{d}{dx}\sin x$ (3)

$$4 - 5\frac{dy}{dx} = \cos x$$

$$5\frac{dy}{dx} = 4 - \cos x$$

$$\frac{dy}{dx} = \frac{4-\cos x}{5}$$

ii) Area of the circle $A=\pi r^2$ (3)

then $\frac{dA}{dr}=2\pi a=2.\pi 3=6\pi cm^2/s$

19\. i) $\frac{x}{(x+1)(x+2)}=\frac{A}{x+1}+\frac{B}{x+2}$ (3)

$= \frac{A(x+2)+B(x+1)}{(x+1)(x+2)}$

Now, $x=A(x+2)+B(x+1)$

Put x = - 1 ,then $-1=A(1)$, $A=-1$

Put x = - 2 , then $-2=B(-1)$, $B=2$

$\frac{x}{(x+1)(x+2)}=\frac{-1}{x+1}+\frac{2}{x+2}$

Now, $\int \frac{x}{(x+1)(x+2)}dx=\int \frac{-1}{x+1}dx+\int \frac{2}{x+2}dx$

= - log (x+1) + 2 log (x+2) + C

$= \log\left|\frac{(x+2)^2}{x+1}\right|+C$

ii) Let I = $\int_0^{\frac{\pi}{2}} \cos^2 dx$ (3)

Therefore by property, $\int_0^a f(x)dx=\int_0^a f(a-x)dx$

$\int_0^{\frac{\pi}{2}} \left(\cos\left(\frac{\pi}{2}-x\right)\right)^2 dx=\int_0^{\frac{\pi}{2}} \sin^2 x\, dx$ = I

Adding them 2I = $\int_0^{\frac{\pi}{2}} \cos^2 x\, dx+\int_0^{\frac{\pi}{2}} \sin^2 x\, dx$

$= \int_0^{\frac{\pi}{2}} (\cos^2 x+\sin^2 x)dx$

$= \int_0^{\frac{\pi}{2}} 1\, dx$

$= [x]_0^{\frac{\pi}{2}}$

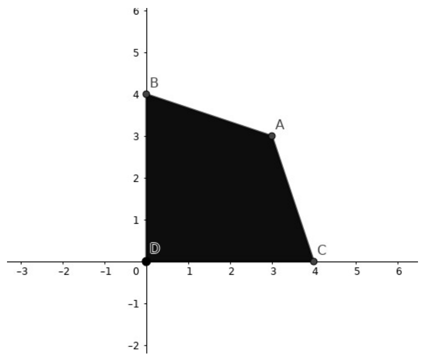
2I = $\frac{\pi}{2}$

I = $\frac{\pi}{4}$

20\. $3x+y\le 12$ $x+3y\le 12$ (6)

x	y
0	12
4	0

x	y
0	4
12	0



Corner point	Z = 17.5 x + 7 y
(0,0)	0
(0,4)	28
(4,0)	70
(3,3)	73.5

Maximum of Z is 73.5 at (3,3).

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